TCAD Avalanche Models

Group meeting
19-05-2020

Preview

- **Wolff** :
  - First theoretical explanation of charge multiplication.
  - Solved quasi-Maxwellian equation with spherically symmetric distribution function.
  - Ionization coefficient valid for **high field** strengths :

$$\boldsymbol{\alpha}(\epsilon) = \mathbf{a}\ \mathbf{exp}(-\mathbf{b}/\epsilon^2)$$

- **Shockley:**
  - A simple statistical model with three adjustable parameters
  - Spike distribution function.
  - Ionization coefficients valid for **low field** strengths :

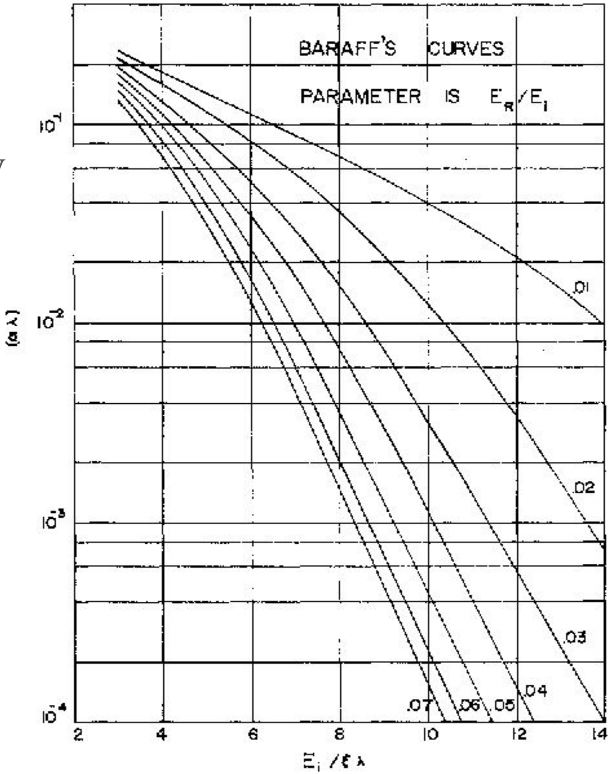
$$\boldsymbol{\alpha}(\epsilon) = (\mathbf{q}\epsilon/\mathbf{r}\mathbf{E}_R)\mathbf{exp}(-\mathbf{E}_i/\mathbf{q}\mathbf{L}_R\epsilon)$$

Preview

- **Baraff** :
  - Distribution function corresponds to Wolff's spherically symmetric part and Shockley's spike with constant mean free path.
  - Similarities in slope to Wolff's theory for higher fields and Shockley's theory for lower fields.
- **Crowell & Sze** : modified Baraff's results
  - introduced parameters - average phonon energy lost per scattering and temperature.

$$\langle E_R \rangle = E_R \, tgh(E_R/2kT)$$

- Later **Keldysh** confirmed Baraff's results.

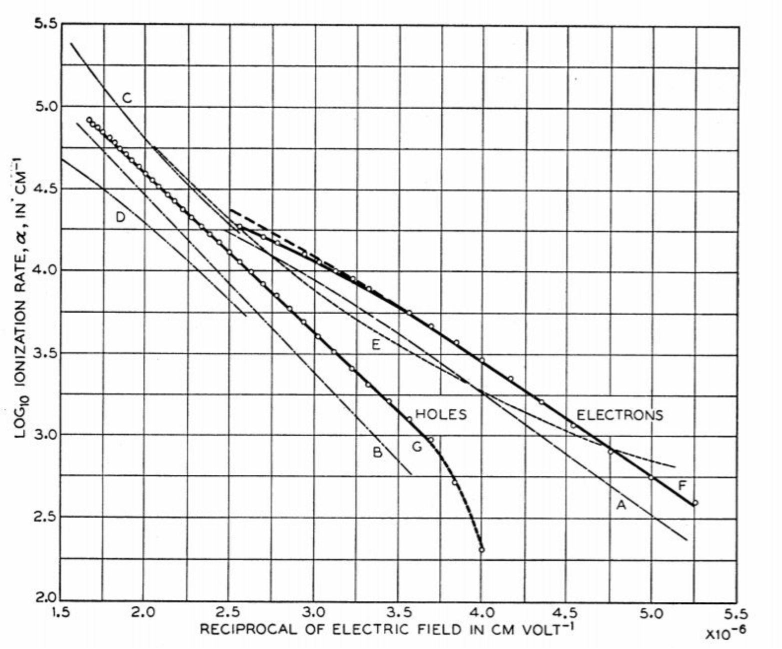


TCAD Avalanche Models

- Van Overstraeten and de Man Model
- Okuto and Crowell Model
- Lackner Model
- UNIBO (University of Bologna) Model
- New UNIBO Model

Chynoweth Law

- Chynoweth's experimental ionization rates agreed well with measurement of ionization rates in gases : **$\alpha = a \exp(-b/\epsilon)$**
- Observed linearity in ln**α** against E^{-1} graph over wide range of fields.
- Shockley and Van Overstraeten validated this law in the electric field range from 1.75 ×10^5 - 6×10^5 Vcm^{-1} (300K).



(A & B correspond to Chynoweth law)

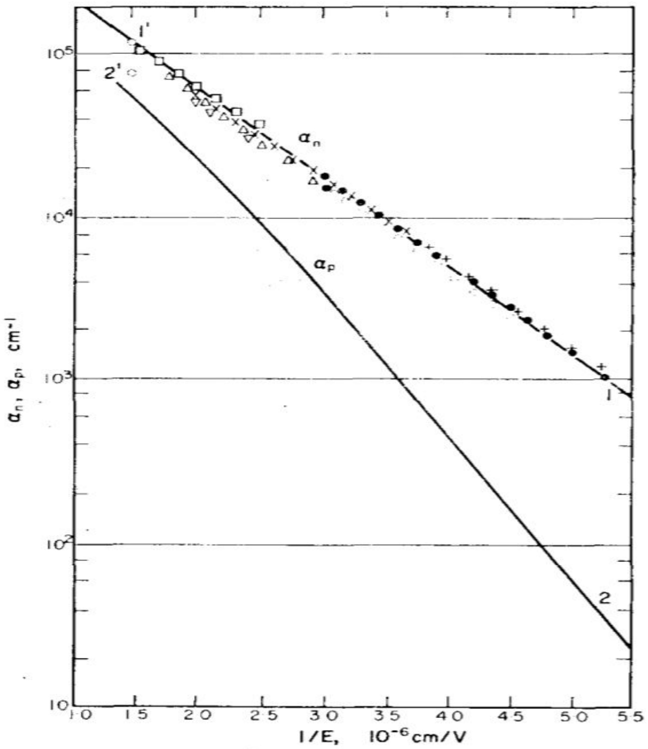
Van Overstraeten Model

- Used Chynoweth's law for fitting the data and extracting the model parameters :

$$\alpha(\epsilon_{ava}) = \gamma a \exp(- \gamma b/\epsilon_{ava}) \; ;$$

$$\gamma = \tanh(\hbar\omega_{op}/2kT_0)/\tanh(\hbar\omega_{op}/2kT)$$

- Valid in the field range of $1.75\times10^5 \leq \epsilon \leq 6\times10^5$ Vcm^{-1} .
- Different values of parameters **a & b** in low and high range of fields.
- Almost same value of **a & b** for 7 different diodes used - made good approximation of Chynoweth's law.



(Symbols represent particular diode)

Van Overstraeten Model

Symbol	Parameter name	Electrons	Holes	Valid range of electric field	Unit
a	a(low)	7.03×10^5	1.582×10^6	$1.75{\times}10^5\,\mathrm{Vcm}^{-1}$ to E_0	cm^{-1}
	a(high)	7.03×10^5	6.71×10^5	E_0 to $6{\times}10^5\,\mathrm{Vcm}^{-1}$	
b	b(low)	1.231×10^6	2.036×10^6	$1.75{\times}10^5\,\mathrm{Vcm}^{-1}$ to E_0	V/cm
	b(high)	1.231×10^6	1.693×10^6	E_0 to $6{\times}10^5\,\mathrm{Vcm}^{-1}$	
E_0	E0	4×10^5	4×10^5		V/cm
$\hbar\omega_{op}$	hbarOmega	0.063	0.063		eV
λ	lambda	62×10^{-8}	45×10^{-8}		cm
β	beta(low)	0.678925	0.815009	$1.75{\times}10^5\,\mathrm{Vcm}^{-1}$ to E_0	1
	beta (high)	0.678925	0.677706	E_0 to $6{\times}10^5\,\mathrm{Vcm}^{-1}$	

Okuto & Crowell Model

- Empirical model based on Baraff's theoretical model :

$$\alpha(\epsilon_{ava}) = a\ [1+c(T-T_0)]\ \epsilon^{\gamma}_{ava}\ \exp(-(b[1+d(T-T_0)] / \epsilon_{ava})^{\delta})\ ;\quad T_0=300K$$

- Energy conservation conditions applied.
- Applicable in the field range of 10^5-10^6 Vcm^{-1} .
- Applied pseudo-local approximation and predicted the existing data with good accuracy.
- "Exact" non-localized approximation considered boundary regions (p-i-n junction).
    - Zero ionization coefficient in boundary dark spaces (p-i & i-n)
    - Constant and close to the values of previous approximation in rest of the region.

Okuto & Crowell Model

Symbol	Parameter name	Electrons	Holes	Unit
a	a	0.426	0.243	V^{-1}
b	b	4.81×10^{5}	6.53×10^{5}	V/cm
c	c	3.05×10^{-4}	5.35×10^{-4}	K^{-1}
d	d	6.86×10^{-4}	5.67×10^{-4}	K^{-1}
γ	gamma	1	1	1
δ	delta	2	2	1
λ	lambda	62×10^{-8}	45×10^{-8}	cm
β	beta	0.265283	0.261395	1

Lackner Model

- New theory on pseudo-local ionization probability model for field correction of Chynoweth law :

$$\alpha_v(\epsilon_{ava}) = (\gamma a_v / Z) \exp(- \gamma b_v/\epsilon_{ava}) \; ; \quad \text{where } v = n,p$$

$$Z = 1 + (\gamma b_n/\epsilon_{ava}) \exp(-\gamma b_n/\epsilon_{ava}) + \gamma b_p/\epsilon_{ava} \exp(-\gamma b_p/\epsilon_{ava})$$

$$\gamma = \tanh(\hbar\omega_{op}/2kT_0)/ \tanh(\hbar\omega_{op}/2kT)$$

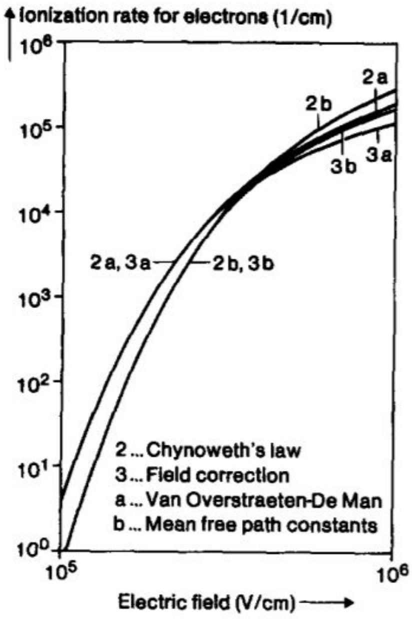
- Introduced Z parameter in Chynoweth's law.
- Valid in the field range 2×10^5 - 6×10^5 Vcm^{-1}.
- Unlike Van Overstraeten and de Man model, same values of parameters **a & b** in low and high field regions.

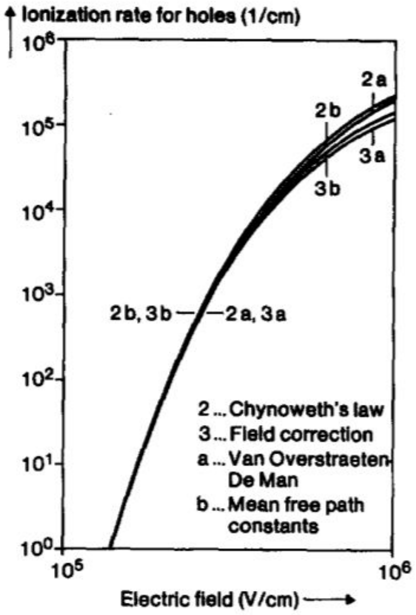
Lackner Model

Symbol	Parameter name	Electrons	Holes	Unit
a	a	1.316×10^{6}	1.818×10^{6}	cm^{-1}
b	b	1.474×10^{6}	2.036×10^{6}	V/cm
$\hbar\omega_{op}$	hbarOmega	0.063	0.063	eV
λ	lambda	62×10^{-8}	45×10^{-8}	cm
β	beta	0.812945	0.815009	1

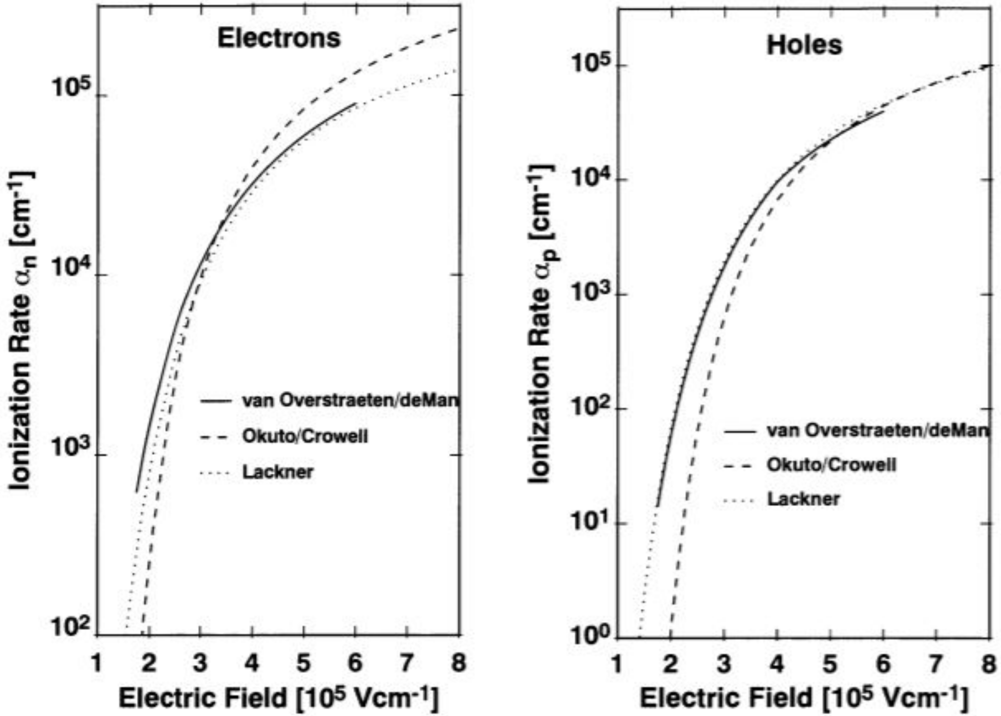
Lackner Model

- Field correction matched well with Chynoweth's law (Z=1) for field values < 4×10^5 Vcm^{-1}.
- Deviation (Z≠1) in higher range of fields (>4×10^5Vcm^{-1}).
- No validation issue has been provided for field correction approximation.





Comparison of Three Models



UNIBO Model

- Compact model based on impact ionization data generated by Boltzmann solver HARM :

$$\alpha(\epsilon_{ava}, T) = \epsilon_{ava} / (a(T) + b(T) \exp[d(T) / \epsilon_{ava} + c(T)])$$

- Ionization coefficient as a function of field and lattice temperature
- Developed for an extended temperature range (300K-675K) and low electric fields(4×10^4 - $5\times10^5 Vcm^{-1}$).
- Observed the contribution of non-equilibrium Auger generation at high temperatures - differentiate this model from other standard models.

New UNIBO Model

- Modification of compact UNIBO model by extending the temperature range between 300-773K :

$$\alpha(\epsilon_{ava},T) = \epsilon_{ava} / (a(T)+b(T)\exp[d(T) / \epsilon_{ava}+c(T)])$$

- Likewise UNIBO, New UNIBO model developed for low electric field range.
- Theoretically based on UNIBO theory
    - Different method of solving the parameters, so, resulted different values.

Conclusions

- Van Overstraeten and Lackner model based on Chynoweth law, but used different parameters
    - reliable for low-field regions (Shockley's low field theory).
- Okuto and Crowell model valid for whole range of fields (Baraff's theory)..
- UNIBO and New UNIBO models showed their reliability for high temperature ranges and low electric fields(down to $4\times10^{4}Vcm^{-1}$).
- All models except Okuto & Crowell typically used for self-heating power devices and ESD-protection structures.
- Our focus is on thin p-n junctions with moderate and high field regions - **Okuto and Crowell**.