Some notes on transformers for MMCs

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September 2008

Here some thoughts on adding a step up transformer to MMC designs are summarized. These are driven by the fact that most MMCs with low inductance meanders suffer from the fact that their noise level is dominated by the SQUIDs used rather than by the more fundamental noise due to thermodynamic fluctuations. A step up transformer might help to improve this situation by enhancing the flux coupling form the meander to the SQUID.

This text should be understod as describing the evolution of the thoughts NOT just giving a final picture but rather the stream of ideas that lead to it.

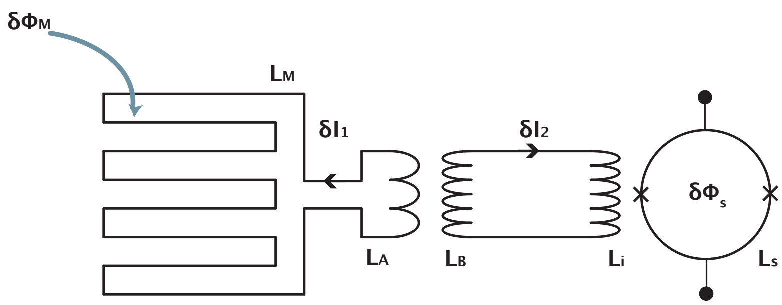

Figure 1: Schematic of MMC with superconducting trafo.

Considering the scheme of the meander with inductance L_M connected to a SQUID with inductance L_s and input inductance of L_i via a superconducting flux transformer with inductances L_A and L_B, as depicted in Figure 1, the following three equations should hold:

$$\delta\Phi_\mathrm{M} - \delta I_1(L_\mathrm{M} + L_\mathrm{A}) + \delta I_2 M_\mathrm{AB} = 0 \tag{1}$$

$$\delta I_1 M_\mathrm{AB} + \delta I_2(L_\mathrm{B} + L_\mathrm{i}) = 0 \tag{2}$$

$$\delta\Phi_\mathrm{s} = \delta I_2 M_\mathrm{is} \tag{3}$$

rearranging equation 2 gives

$$\delta I_1 = \frac{L_\mathrm{B} + L_\mathrm{i}}{M_\mathrm{AB}}\delta I_2. \tag{4}$$

Putting this into (1) and using the mutual inductances $M_{\mathrm{AB}} = \sqrt{L_{\mathrm{A}} L_{\mathrm{B}}}$ and $M_{\mathrm{is}} = \sqrt{L_{\mathrm{i}} L_{\mathrm{s}}}$ this leads to the following expression relating the flux change in the meander to the flux change in the SQUID

$$\frac{\delta\Phi_{\mathrm{s}}}{\delta\Phi_{\mathrm{M}}} = \frac{\sqrt{L_{\mathrm{i}} L_{\mathrm{s}}}}{\frac{(L_{\mathrm{M}}+L_{\mathrm{A}})(L_{\mathrm{B}}+L_{\mathrm{i}})}{\sqrt{L_{\mathrm{A}} L_{\mathrm{B}}}} - \sqrt{L_{\mathrm{A}} L_{\mathrm{B}}}} \tag{5}$$

In Figure 2 a plot of the "flux-coupling-coefficient" $\delta\Phi_{\mathrm{s}}/\delta\Phi_{\mathrm{M}}$ for varying inductances L_{A} and L_{B} is shown for a given meander inductance $L_{\mathrm{M}} = 1\mathrm{nH}$ and a given SQUID input inductance $L_{\mathrm{i}} = 1.8\mathrm{nH}$ representing the "small" PTB SQUID.

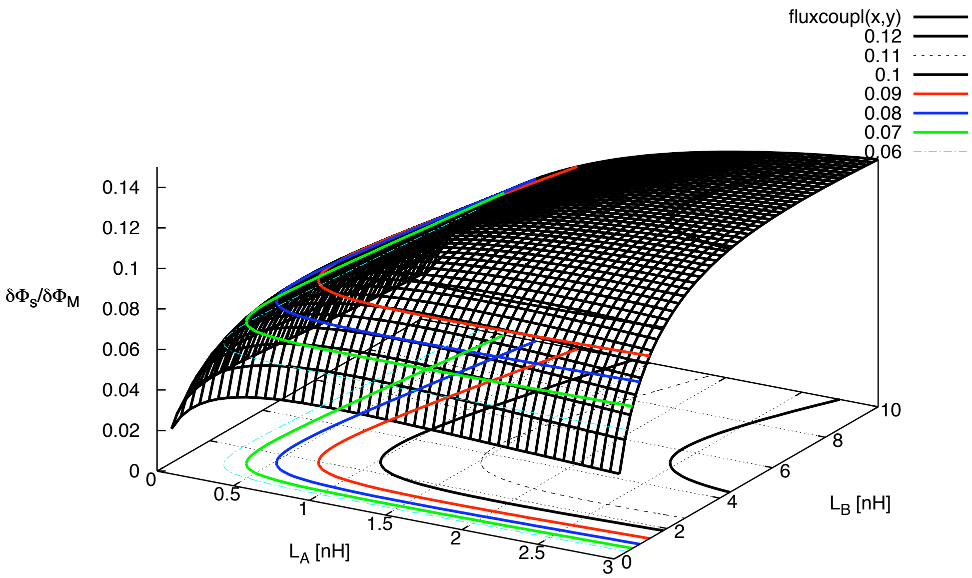

Figure 2: Coupling $\delta\Phi_{\mathrm{s}}/\delta\Phi_{\mathrm{M}}$ of the flux in the meander to flux in the SQUID as a function of the two transformer inductances. Here the meander inductance is fixed to $L_{\mathrm{M}} = 1\mathrm{nH}$ and the SQUID's input inductance is $L_{\mathrm{s}} = 1.8\mathrm{nH}$.

This shows that the coupling is getter better the bigger the inductances L_{A} and L_{B} are, but why? To understand this, let's have a look if there is any obvious dependence of one of those inductances on the overall coupling, when we keep the other one fixed (also with the meander inductance fixed to one value).

Figure 3 shows the dependance of the coupling-coefficient on the primary transformer inductance L_{A} for a given secondary transformer inductance that is matched to the SQUID input inductance, i.e. $L_{\mathrm{B}} = L_{\mathrm{i}}$, and a given meander inductance. Furthermore, it shows the same for $L_{\mathrm{A}} = 2L_{\mathrm{M}}$ and varying L_{B}.

So it seems there is a maximum for a certain ratio of L_{A} compared to L_{M} with fixed L_{B} and the same is true for a ratio of L_{B} compared to L_{i} with fixed L_{A}, let's have a closer look to that by expressing the inductance of the primary side of the transformer in units of the meander inductance $L_{\mathrm{A}} = aL_{\mathrm{M}}$ and setting the secondary side of the transformer

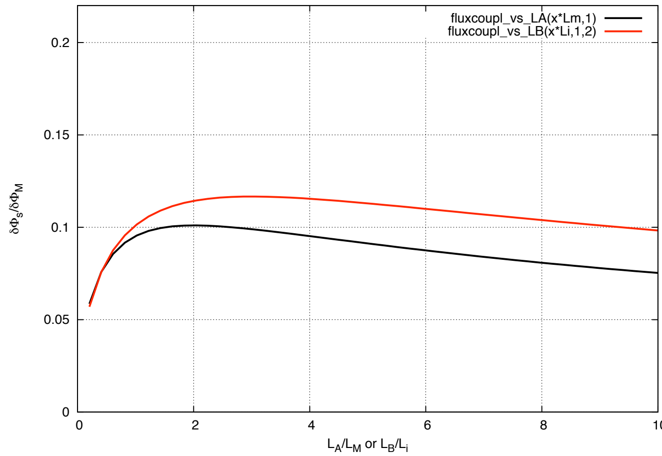

Figure 3: Coupling to the SQUID with varying L_A in units of L_M with fixed $L_\mathrm{B} = L_\mathrm{i}$ (black) and with varying L_B in units of L_i with fixed $L_\mathrm{A} = 2L_\mathrm{M}$ (red). In both cases $L_\mathrm{M} = 1nH$.

equal to the input coil inductance of the SQUID, the flux-coupling-coefficient can be written as follows:

$$\frac{\delta\Phi_\mathrm{s}}{\delta\Phi_\mathrm{M}} = \frac{\sqrt{a}}{(a+2)}\sqrt{\frac{L_\mathrm{s}}{L_\mathrm{M}}}, \tag{6}$$

this shows a maximum for $a = 2$, leading to:

$$\frac{\delta\Phi_\mathrm{s}}{\delta\Phi_\mathrm{M}} = 2^{-\frac{3}{2}}\sqrt{\frac{L_\mathrm{s}}{L_\mathrm{M}}}. \tag{7}$$

So some systematics can be found, but taking this idea to the next step the flux-coupling-coefficient can even be parameterized further. Introducing not only $L_\mathrm{A} = aL_\mathrm{M}$ but also $L_\mathrm{B} = bL_\mathrm{i}$ the expression is even more general:

$$\frac{\delta\Phi_\mathrm{s}}{\delta\Phi_\mathrm{M}} = \frac{\sqrt{a \cdot b}}{(a+b+1)}\sqrt{\frac{L_\mathrm{s}}{L_\mathrm{M}}}, \tag{8}$$

The flux-coupling-coefficient is shown in Figure 4 for several scenarios (described in the caption) showing the same overall behaviour that can also be seen in Formula (8):

- The coupling will get better with smaller meander inductance. This might need to get fixed by including some stray inductance L_stray, e.g. by adding some amount to L_M, this will shift the curve a bit (also displayed with $L_\mathrm{stray} = 100\,\mathrm{pH}$).
- The larger a and b, the better the coupling.
- It can be shown that for given a best coupling will be reached for $a = b$

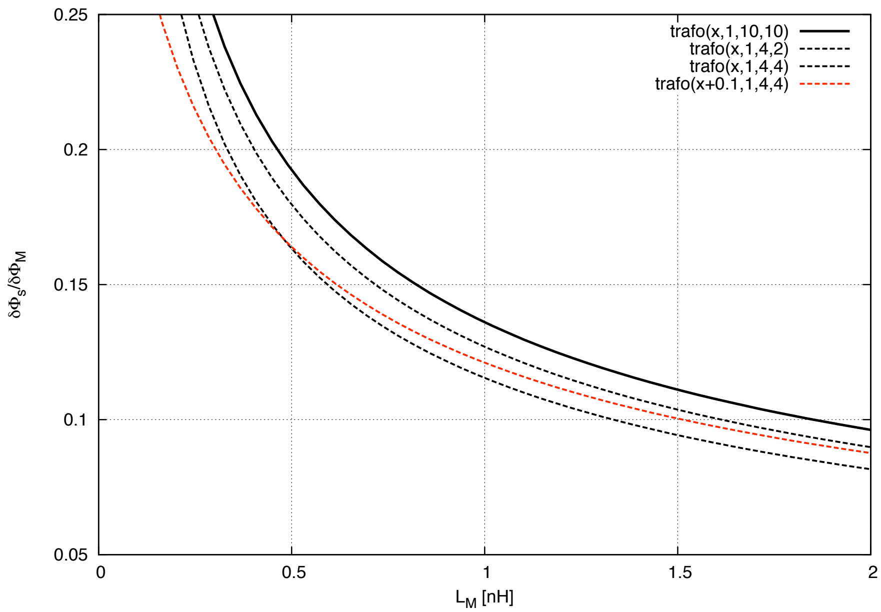

Figure 4: Comparing flux-coupling-coefficient for different conditions. **1.:** $a = b = 10$; **2.:** $a \neq b$ with $a = 4$, $b = 2$; **3.:** $a = b = 4$ and **4.:** $a = b = 4$ plus some $L_{\text{stray}} = 0.1\,nH$

Especially the fact that the coupling coefficient can be improved infinitely with large a and b calls for another improvement of the model. So far the transformer was assumed to be perfect and an actual efficiency of the transformer was neglected. Up to now the theoretical mutual inductance of $M_{\text{AB}} = \sqrt{L_{\text{A}} L_{\text{B}}}$ was used, a correction factor k will be needed to improve this model

$$M_{\text{AB}} = k\sqrt{L_{\text{A}} L_{\text{B}}}.$$

Simulations show that this factor is expected to be between $0.85 < k \leq 1$. This factor depends on some constraints of the actual microstructures used in the fabrication, e.g. linewidth of the structures, the thickness of insulation layers etc. . .

In the following I will try to introduce this factor k in the flux-coupling-coefficient for further analysis of it

$$\frac{\delta\Phi_{\text{s}}}{\delta\Phi_{\text{M}}} = \frac{M_{\text{is}}}{\frac{(L_{\text{M}}+aL_{\text{M}})(bL_{\text{i}}+L_{\text{i}})}{k\sqrt{ab}\sqrt{L_{\text{M}}L_{\text{i}}}} - k\sqrt{ab}\sqrt{L_{\text{M}}L_{\text{i}}}} \tag{9}$$

$$= k\sqrt{ab}\sqrt{\frac{L_{\text{s}}}{L_{\text{M}}}}\,\frac{1}{a+b+(1-k^2)ab+1} \tag{10}$$

Looking for a maximum for this expression with respect to a and b will help building a transformer that is well matched to the entire MMC setup. This maximum will certainly depend on the quality of transformer one can build, namely on the factor k. As was shown above for a perfect transformer, a should equal b and the larger the two the better the overall flux coupling from the meander to the SQUID. This will change now. Setting the derivatives of (10) with respect to a and b to zero yields to

$$a = \frac{1+b}{1+(1-k^2)b} \tag{11}$$

and

$$b = \frac{1+a}{1+(1-k^2)a}, \tag{12}$$

respectively. This shows that for best coupling a should still equal b, but that the maximum will now strongly depend on the efficiency k of the transformer (illustrated in Figure 5.

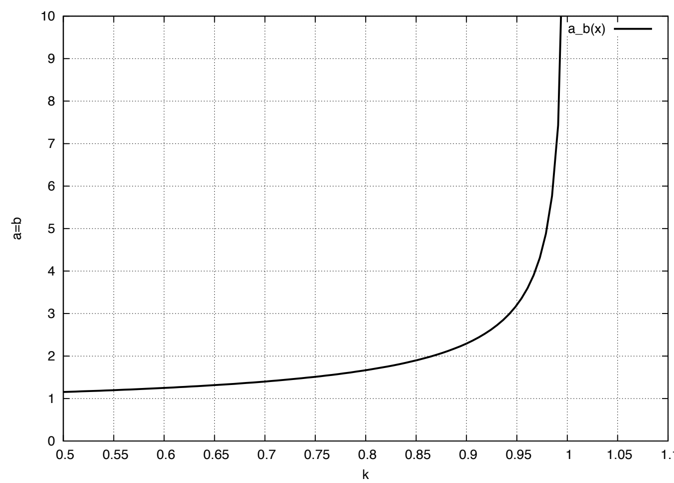

Figure 5: Dependence of the a and b on the transformer efficiency k.

Example: Considering a transformer with $M_{\mathrm{AB}} = k\sqrt{L_{\mathrm{A}} L_{\mathrm{B}}}$ with a k of 0.95, the best flux coupling will be achieved for

$$a = b = \sqrt{\frac{1}{1-0.95^2}} = 3.2. \tag{13}$$

i.e. the primary side of the transformer should have 3.2 times the inductance of the meander, whereas the secondary side of the transformer should have an inductance that is 3.2 times the inductance of the input coil of the SQUID. In the case of transformer efficiency of $k = 0.85$ the best coupling will be reached for $a = b = 1.9$.

In Figure 6 a comparison of the flux-coupling-coefficient for varying meander inductance in the following different scenarios is shown:

- Setup of direct coupling to the SQUID of a gradiometric design (with L_{stray}=0.8 nH)
- Setup of direct coupling to the SQUID (with three different stray inductances assumed (L_{stray}=0.3 nH and L_{stray}=0.8 nH and L_{stray}=1.2 nH)

- Setup with a perfect transformer
- Setup with a transformer with efficiency $M_{\mathrm{AB}} = k\sqrt{L_{\mathrm{A}} L_{\mathrm{B}}}$ with $k = 0.98$ and matched inductances with $a = b = 5.0$
- Setup with a transformer with efficiency $M_{\mathrm{AB}} = k\sqrt{L_{\mathrm{A}} L_{\mathrm{B}}}$ with $k = 0.95$ and matched inductances with $a = b = 3.2$
- Setup with a transformer with efficiency $M_{\mathrm{AB}} = k\sqrt{L_{\mathrm{A}} L_{\mathrm{B}}}$ with $k = 0.85$ and matched inductances with $a = b = 1.9$

This helps to compare with setups currently used and gives an idea for what size of meanders the described step up transformers will improve the flux coupling and thus the achievable energy resolution.

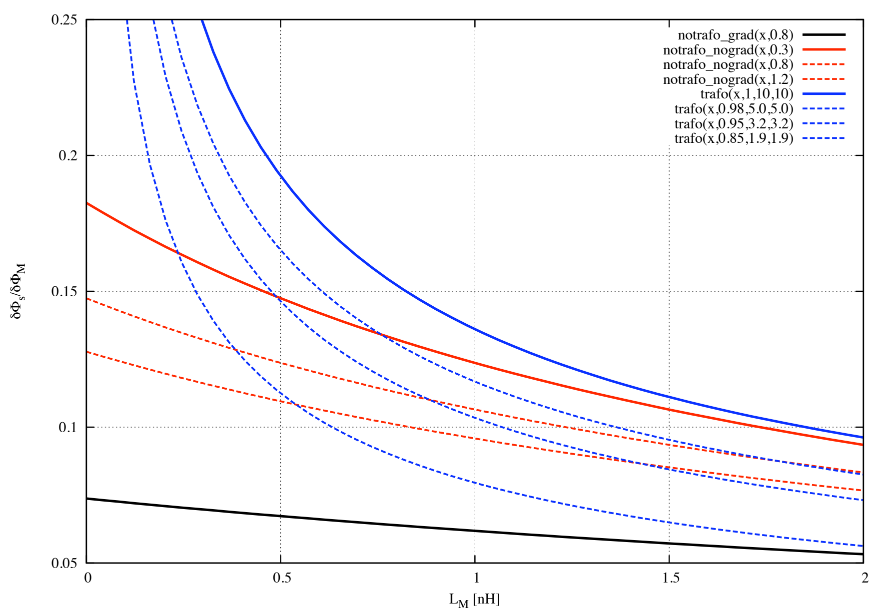

Figure 6: Comparing the coupling to the SQUID of a setup with and without a transformer for several cases described in detail within the text.

Note: In the case without a transformer the following coupling expressions were used:

$$\frac{\delta\Phi_{\mathrm{s}}}{\delta\Phi_{\mathrm{M}}} = \frac{M_{\mathrm{is}}}{L_{\mathrm{M}} + 2(L_{\mathrm{i}} + L_{\mathrm{stray}})} \tag{14}$$

$$\frac{\delta\Phi_{\mathrm{s}}}{\delta\Phi_{\mathrm{M}}} = \frac{M_{\mathrm{is}}}{L_{\mathrm{M}} + L_{\mathrm{i}} + L_{\mathrm{stray}}} \tag{15}$$

for the cases of a two pixel gradiometric and single pixel design, respectively.

Some further notes on transformers...

Without taking into account stray inductances, the flux-coupling can be written as

$$\frac{\Phi_{\mathrm{s}}}{\Phi_{\mathrm{M}}} = \frac{M_{\mathrm{is}} M_{\mathrm{AB}}}{(L_{\mathrm{M}} + L_{\mathrm{A}})(L_{\mathrm{B}} + L_{\mathrm{i}}) - {M_{\mathrm{AB}}}^2} \tag{16}$$

as seen above.

Taking into account a stray-inductances $L_{\mathrm{S,m}}$ between the meander and the primary side of the transformer and a stray-inductance $L_{\mathrm{S,t}}$ between the secondary side of the transformer and the input coil of the SQUID the coupling can be written as:

$$\frac{\Phi_{\mathrm{s}}}{\Phi_{\mathrm{M}}} = \frac{M_{\mathrm{is}} M_{\mathrm{AB}}}{(L_{\mathrm{M}} + L_{\mathrm{S,m}} + L_{\mathrm{A}})(L_{\mathrm{B}} + L_{\mathrm{S,t}} + L_{\mathrm{i}}) - {M_{\mathrm{AB}}}^2} \tag{17}$$

Introducing a finite coupling and the rewriting the transformers coil inductances as multiples of the meander inductance and the input coil inductance of the SQUID:

$$\frac{\Phi_{\mathrm{s}}}{\Phi_{\mathrm{M}}} = \frac{k \cdot \sqrt{ab} \cdot M_{\mathrm{is}}}{[(1-k^2)ab + a + b + 1]\sqrt{L_{\mathrm{M}} L_{\mathrm{i}}} + (a+1)\sqrt{\frac{L_{\mathrm{M}}}{L_{\mathrm{i}}}} L_{\mathrm{S,t}} + (b+1)\sqrt{\frac{L_{\mathrm{i}}}{L_{\mathrm{M}}}} L_{\mathrm{S,m}} + \frac{L_{\mathrm{S,m}} L_{\mathrm{S,t}}}{\sqrt{L_{\mathrm{M}} L_{\mathrm{i}}}}} \tag{18}$$

Setting all stray-inductances to zero, as well as setting $k = 1$ leads to the formula as seen before:

$$\frac{\Phi_{\mathrm{s}}}{\Phi_{\mathrm{M}}} = \frac{\sqrt{ab} \cdot M_{\mathrm{is}}}{[a + b + 1]\sqrt{L_{\mathrm{M}} L_{\mathrm{i}}}} \tag{19}$$

For $k \neq 1$ we can now analytically maximize equation 18 and for the optimal ratio a/b we obtain:

$$\frac{a}{b} = \frac{\sqrt{L_{\mathrm{M}} L_{\mathrm{i}}} + \sqrt{\frac{L_{\mathrm{i}}}{L_{\mathrm{M}}}} \cdot L_{\mathrm{S,m}}}{\sqrt{L_{\mathrm{M}} L_{\mathrm{i}}} + \sqrt{\frac{L_{\mathrm{M}}}{L_{\mathrm{i}}}} \cdot L_{\mathrm{S,t}}} \tag{20}$$

The coupling is then maximal for

$$a = \sqrt{\frac{(L_{\mathrm{M}} + L_{\mathrm{S,m}})(\sqrt{L_{\mathrm{M}} L_{\mathrm{i}}} + \sqrt{\frac{L_{\mathrm{i}}}{L_{\mathrm{M}}}} L_{\mathrm{S,m}} + \sqrt{\frac{L_{\mathrm{M}}}{L_{\mathrm{i}}}} L_{\mathrm{S,t}} + \frac{L_{\mathrm{S,t}} L_{\mathrm{S,m}}}{\sqrt{L_{\mathrm{M}} L_{\mathrm{i}}}})}{(1-k^2) L_{\mathrm{M}} \left(\sqrt{L_{\mathrm{M}} L_{\mathrm{i}}} + \sqrt{\frac{L_{\mathrm{M}}}{L_{\mathrm{i}}}} L_{\mathrm{S,t}}\right)}} \tag{21}$$

Although it does not look nice it seems to be ok …as it agrees with the results seen before in the the limit of no stray inductances and perfect coupling.

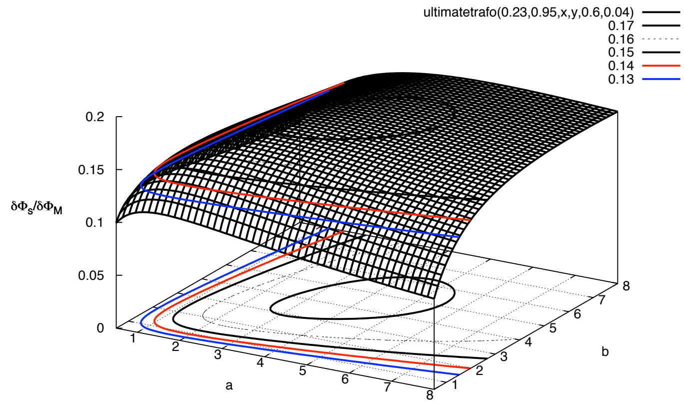

Figure 7: Illustrating the dependance of the flux coupling of a given meander, given SQUID and given strays on the ratios a and b.

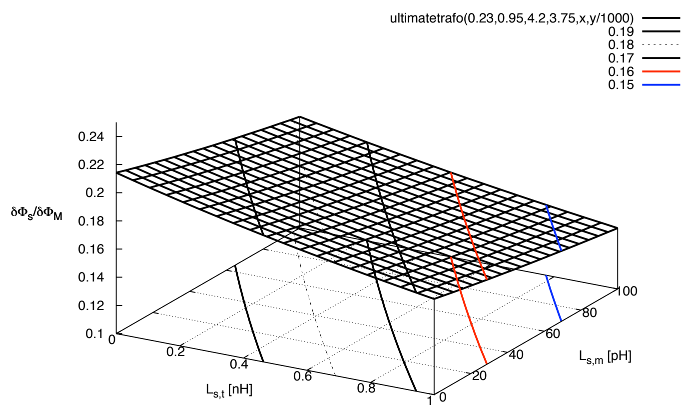

Figure 8: Illustrating the dependance of the flux coupling of a given meander, given SQUID and given transformer inductances on the occurring stray inductances.

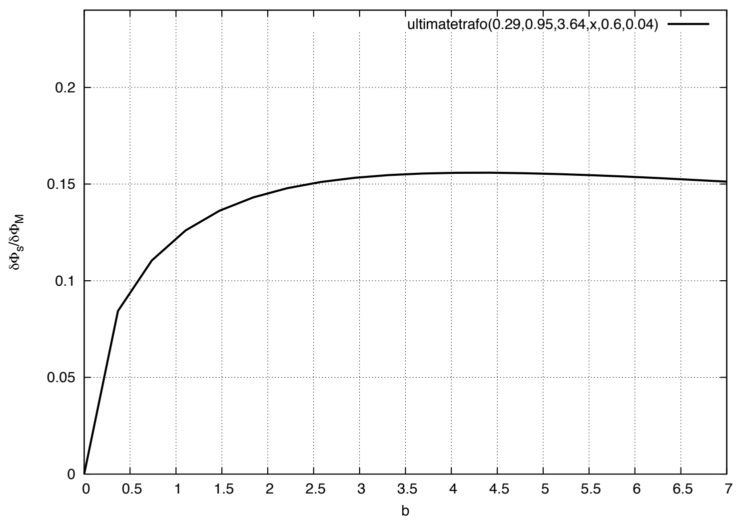

Figure 9: Illustrating the dependance of the flux coupling of a given meander, given SQUID and given primary side of transformer inductances on possible secondary transformer side.