

Standard Curve 10: Measurement Current = 10 µA ±0.05%

T (K)	Voltage	dV/dT (mV/K)	T (K)	Voltage	dV/dT (mV/K)	T (K)	Voltage	dV/dT (mV/K)
1.40	1.69812	-13.1	16.0	1.28527	-18.6	95.0	0.98564	-2.02
1.60	1.69521	-15.9	16.5	1.27607	-18.2	100.0	0.97550	-2.04
1.80	1.69177	-18.4	17.0	1.26702	-18.0	110.0	0.95487	-2.08
2.00	1.68786	-20.7	17.5	1.25810	-17.7	120.0	0.93383	-2.12
2.20	1.68352	-22.7	18.0	1.24928	-17.6	130.0	0.91243	-2.16
2.40	1.67880	-24.4	18.5	1.24053	-17.4	140.0	0.89072	-2.19
2.60	1.67376	-25.9	19.0	1.23184	-17.4	150.0	0.86873	-2.21
2.80	1.66845	-27.1	19.5	1.22314	-17.4	160.0	0.84650	-2.24
3.00	1.66292	-28.1	20.0	1.21440	-17.6	170.0	0.82404	-2.26
3.20	1.65721	-29.0	21.0	1.19645	-18.5	180.0	0.80138	-2.28
3.40	1.65134	-29.8	22.0	1.17705	-20.6	190.0	0.77855	-2.29
3.60	1.64529	-30.7	23.0	1.15558	-21.7	200.0	0.75554	-2.31
3.80	1.63905	-31.6	24.0	1.13598	-15.9	210.0	0.73238	-2.32
4.00	1.63263	-32.7	25.0	1.12463	-7.72	220.0	0.70908	-2.34
4.20	1.62602	-33.6	26.0	1.11896	-4.34	230.0	0.68564	-2.35
4.40	1.61920	-34.6	27.0	1.11517	-3.34	240.0	0.66208	-2.36
4.60	1.61220	-35.4	28.0	1.11212	-2.82	250.0	0.63841	-2.37
4.80	1.60506	-36.0	29.0	1.10945	-2.53	260.0	0.61465	-2.38
5.00	1.59782	-36.5	30.0	1.10702	-2.34	270.0	0.59080	-2.39
5.50	1.57928	-37.6	32.0	1.10263	-2.08	280.0	0.56690	-2.39
6.00	1.56027	-38.4	34.0	1.09864	-1.92	290.0	0.54294	-2.40
6.50	1.54097	-38.7	36.0	1.09490	-1.83	300.0	0.51892	-2.40
7.00	1.52166	-38.4	38.0	1.09131	-1.77	310.0	0.49484	-2.41
7.50	1.50272	-37.3	40.0	1.08781	-1.74	320.0	0.47069	-2.42
8.00	1.48443	-35.8	42.0	1.08436	-1.72	330.0	0.44647	-2.42
8.50	1.46700	-34.0	44.0	1.08093	-1.72	340.0	0.42221	-2.43
9.00	1.45048	-32.1	46.0	1.07748	-1.73	350.0	0.39783	-2.44
9.50	1.43488	-30.3	48.0	1.07402	-1.74	360.0	0.37337	-2.45
10.0	1.42013	-28.7	50.0	1.07053	-1.75	370.0	0.34881	-2.46
10.5	1.40615	-27.2	52.0	1.06700	-1.77	380.0	0.32416	-2.47
11.0	1.39287	-25.9	54.0	1.06346	-1.78	390.0	0.29941	-2.48
11.5	1.38021	-24.8	56.0	1.05988	-1.79	400.0	0.27456	-2.49
12.0	1.36809	-23.7	58.0	1.05629	-1.80	410.0	0.24963	-2.50
12.5	1.35647	-22.8	60.0	1.05267	-1.81	420.0	0.22463	-2.50
13.0	1.34530	-21.9	65.0	1.04353	-1.84	430.0	0.19961	-2.50
13.5	1.33453	-21.2	70.0	1.03425	-1.87	440.0	0.17464	-2.49
14.0	1.32412	-20.5	75.0	1.02482	-1.91	450.0	0.14985	-2.46
14.5	1.31403	-19.9	80.0	1.01525	-1.93	460.0	0.12547	-2.41
15.0	1.30422	-19.4	85.0	1.00552	-1.96	470.0	0.10191	-2.30
15.5	1.29464	-18.9	90.0	0.99565	-1.99	475.0	0.09062	-2.22

Lighter numbers indicate truncated portion of Standard Curve 10 corresponding to the reduced temperature range of DT-471 diode sensors. The 1.4–325 K portion of Curve 10 is applicable to the DT-450 miniature silicon diode sensor.

## POLYNOMIAL REPRESENTATION

Curve 10 can be expressed by a polynomial equation based on the Chebychev polynomials. Four separate ranges are required to accurately describe the curve. Table 1 lists the parameters for these ranges. The polynomials represent Curve 10 on the preceding page with RMS deviations of 10 mK. The Chebychev equation is:

$$T(x) = \sum_{i=0}^{n} a_i t_i(x)$$
 (1)

where T(x) = temperature in kelvin,  $t_i(x)$  = a Chebychev polynomial, and  $a_i$  = the Chebychev coefficient. The parameter x is

a normalized variable given by:

$$X = \frac{(Z - ZL) - (ZU - Z)}{(ZU - ZL)} \tag{2}$$

where Z = voltage and ZL and ZU = lower and upper limit of the voltage over the fit range. The Chebychev polynomials can

$$t_{i+1}(x) = 2xt_i(x) - t_{i-1}(x)$$

be generated from the recursion relation:

$$t_0(x) = 1, t_1(x) = x$$
 (3)

Alternately, these polynomials are given by:

$$t_i(x) = \cos\left[i \times \arccos(x)\right] \tag{4}$$

The use of Chebychev polynomials is no more complicated than the use of the regular power series and they offer significant advantages in the actual fitting process. The first step is to transform the measured voltage into the normalized variable using Equation 2. Equation 1 is then used in combination with equations 3 and 4 to calculate the temperature. Programs 1 and 2 provide sample BASIC subroutines which will take the voltage and return the temperature T calculated from Chebychev fits. The subroutines assume the values ZL and ZU have been input along with the degree of the fit. The Chebychev coefficients are also assumed to be in any array A(0), A(1),..., A(i<sub>degree</sub>).

An interesting property of the Chebychev fits is evident in the form of the Chebychev polynomial given in Equation 4. No term in Equation 1 will be greater than the absolute value of the coefficient. This property makes it easy to determine the contribution of each term to the temperature calculation and where to truncate the series if full accuracy is not required.

```
FUNCTION Chebychev (Z as double) as double
REM Evaluation of Chebychev series
   X=((Z-ZL)-(ZU-Z))/(ZU-ZL)
   Tc(0)=1
   Tc(1)=X
   T=A(0)+A(1)*X
   FOR I=2 to Ubound(A())
        Tc(I)=2*X*Tc(I-1)-Tc(I-2)
        T=T+A(I)*Tc(I)
   NEXT I
   Chebychev=T
END FUNCTION
```

**Program 1**. BASIC subroutine for evaluating the temperature  $\mathcal{T}$  from the Chebychev series using Equations (1) and (3). An array  $\mathcal{T}_{c}$  ( $i_{degree}$ ) should be dimensioned. See text for details.

```
FUNCTION Chebychev (Z as double) as double REM Evaluation of Chebychev series X = ((Z-ZL) - (ZU-Z)) / (ZU-ZL)
T = 0
FOR I = 0 to Ubound(A())
T = T + A(I) * COS(I * ARCCOS(X))
NEXT I
Chebychev=T
END FUNCTION

NOTE: arccos(X) = \frac{\pi}{2} - arctan\left[\frac{X}{\sqrt{1-X^2}}\right]
```

**Program 2**. BASIC subroutine for evaluating the temperature  $\mathcal{T}$  from the Chebychev series using Equations (1) and (4). Double precision calculations are recommended.

**Table 1. Chebychev Fit Coefficients** 

2.0 K to 12.0 K	12.0 K to 24.5 K	24.5 K to 100.0 K	100 K to 475 K	
ZL = 1.32412	ZL = 1.11732	ZL = 0.923174	ZL = 0.079767	
ZU = 1.69812	ZU = 1.42013	ZU = 1.13935	ZU = 0.999614	
A(0) = 7.556358	A(0) = 17.304227	A(0) = 71.818025	A(0) = 287.756797	
A(1) = -5.917261	A(1) = -7.894688	A(1) = -53.799888	A(1) = -194.144823	
A(2) = 0.237238	A(2) = 0.453442	A(2) = 1.669931	A(2) = -3.837903	
A(3) = -0.334636	A(3) = 0.002243	A(3) = 2.314228	A(3) = -1.318325	
A(4) = -0.058642	A(4) = 0.158036	A(4) = 1.566635	A(4) = -0.109120	
A(5) = -0.019929	A(5) = -0.193093	A(5) = 0.723026	A(5) = -0.393265	
A(6) = -0.020715	A(6) = 0.155717	A(6) = -0.149503	A(6) = 0.146911	
A(7) = -0.014814	A(7) = -0.085185	A(7) = 0.046876	A(7) = -0.111192	
A(8) = -0.008789	A(8) = 0.078550	A(8) = -0.388555	A(8) = 0.028877	
A(9) = -0.008554	A(9) = -0.018312	A(9) = 0.056889	A(9) = -0.029286	
	A(10) = 0.039255	A(10) = -0.116823	A(10) = 0.015619	
		A(11) = 0.058580		